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Characterization of multidimensional stochastic orders using multidimensional quantile functions

The quantile function is one of the most frequently used characteristics of the probability distribution on $\mathbb{R}$. In particular, in the description of some one-dimensional stochastic orders - *usual stochastic order*, *hazard rate order*, *reversed hazard order* or *likelihood ratio order* - one can use the quantile function of one of the compared distributions, composing it with the cumulative distribution function of the second one[1].

Due to the fact that the cumulative distribution function of the multidimensional probability distribution cannot be a bijection, there is not a natural version of a quantile function in this case. However, several multidimensional quantile structures are known, and each of them attempts to transfer a property of a quantile function to a multidimensional case, depending on what the object will be used for. In this work one of the concepts of multidimensional quantiles derived from Einmahl and Mason[2] was discussed, and then it was used to describe multidimensional stochastic orders: *lower* and *upper orthant order*, *usual stochastic order*, *weak reverse hazard order*, *weak hazard rate order* and *likelihood ratio order*.

The work consists of six chapters. In the first one, known results concerning the quantile function understood as a generalized inverse of the cumulative distribution function as well as the survival function were ordered and the characterization of the previously mentioned one-dimensional stochastic orders was presented. Then, a version of the Einmahl and Mason quantiles, useful for describing the multidimensional stochastic orders mentioned formerly, was presented. The second chapter contains considerations about selected properties of introduced multidimensional quantile functions. The third chapter discusses the relationships between quantile functions and multidimensional stochastic orders, similar to those that for the one-dimensional case were presented in the first chapter. The fourth chapter contains the implementation of the said results for distributions given by Archimedean copulas. In the fifth chapter, the concept of ordering two pairs of distributions, considered by Lehmann and Rojo, in which one of pairs is more distant from each other than the other, and metrics on distribution spaces reflecting this order, were extended to the multidimensional case. In the sixth chapter, one can find the construction of empirical multidimensional quantile functions and theorems concerning their convergence to previously considered theoretical functions.

[1] E. L. Lehmann, J. Rojo (1992), Invariant directional orderings, *Annals of Statistics*, 20, pp. 2100-2110

[2] J. H. J. Einmahl, D. M. Mason (1992), Generalized quantile process, *Annals of Statistics*, 20, pp. 1062-1078