Exit problems for Lévy type models and their applications

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Abstract

The dissertation considers the exit problems for two classes of stochastic processes derived from the class of the spectrally negative Lévy processes and, in particular, the application of these problems to the theory of ruin and the problem of optimal dividend payment. The work consists of an introduction, three chapters and a bibliography.

The first chapter presents the definitions needed in the remaining parts of the dissertation. Additionally, the problem of optimal dividend payment was introduced with a discussion about the solution to this problem in the case of spectrally negative Lévy processes.

The second chapter considers the solution to the optimal dividend payment problem in the model, which assumes that the risk process has refraction at zero. The process on which this chapter is based is the so-called refracted Lévy process. It is a strong solution to the following stochastic differential equation

$$dR_t = dX_t - \delta \mathbf{1}_{\{R_t > 0\}} dt,$$

where $\delta > 0$, X is the spectrally negative Lévy process and $R_0 = X_0$. Furthermore, in this model, we assume that the moment of bankruptcy is defined by the moment of Parisian ruin, which distinguishes technical default from an actual bankruptcy. Namely, bankruptcy occurs when the risk process stays below zero for longer than the fixed time $r > 0$. In addition, a fixed transaction cost $\beta > 0$ is associated with each dividend payment. The last assumption makes it impossible to use the barrier strategy, a frequent research object in the optimality of dividend payment strategies. Instead of this strategy, we consider an impulse strategy. Namely, we assume that after exceeding the level c_2, the process is lowered to the level c_1, and a dividend of $c_2 - c_1 - \beta$ is paid, i.e. reduced by the transaction cost. In the language of scale functions, we have shown sufficient conditions for optimising this strategy and numerical results illustrating the optimal level of (c_1, c_2) barriers. In addition, we presented analytical formulas for the new scale functions for refracted processes where X is a linear Brownian motion or a Crámer-Lundberg process with exponential claims.

In the third chapter, we consider the exit problems for Markov-additive processes. These are two-dimensional stochastic processes of the (X, J) form, in which the X component is responsible for the process's position (additive part). In contrast, the second component J is responsible for the random environment. The J process is a Markov process with a finite state space $E = \{1, 2, 3, ..., N\}$. On the other hand, when the J process is in the $i \in E$ state, the X process behaves (in increments) like some spectrally negative Lévy process X^i. Additionally, we consider killing stochastic processes with the intensity ω being a bounded and non-negative function depending on the location of the X process and the state of the J process. In other words, the following stopping time is defined as

$$\tau_\omega := \inf\{t > 0 : \int_0^t \omega_{J_s}(X_s) ds > e_1\},$$

where e_1 is an independent exponential variable with parameter 1. Then, we say that the process is killed when $t > \tau_\omega$. When $\omega \equiv q > 0$, we get classical exponential killing. For the model defined in this way, we represent the exit problems in the language of the so-called ω-scale functions, which are matrix-valued. This is an important step, as many practical problems can be expressed in the language of these functions. In particular, this has been illustrated by the value function's representation in the optimal dividend payment problem in the so-called Omega model. In this model, the moment of ruin is defined as

$$\tau_\omega^d := \inf\{t > 0 : \int_0^t \omega_{J_s}(X_s)ds > e_1 \vee X_t < -d\},$$

where (usually) $d > 0$. It was also assumed that in this model, the ω function only takes positive values in a particular interval. This assumption allows the ω function to be interpreted as a function of the penalty for being in the so-called *red zone*. Finally, note that when the X process goes below the $-d$ level, an immediate ruin occurs, which allows for some limitation of the X process value at the time of bankruptcy.

Moreover, some examples of the ω-scale functions are presented for different selections of the ω function where (X, J) is a Markov-modulated Brownian motion, which is an analogue to the classical linear Brownian motion.