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Drift change detection in models based on Lévy processes with applications to mortality analysis

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Abstract

The dissertation studies optimal detection of a drift change in models in which the underlying processes are Lévy processes. In general, the main considered process $X = (X_t)_{t\geq 0}$ can be defined as

$$X_t = \begin{cases} X_t^{(1)}, & t < \theta, \\ X_\theta^{(1)} + X_{t-\theta}^{(2)}, & t \geq \theta, \end{cases}$$

where both $X^{(1)} = (X_t^{(1)})_{t\geq 0}$ and $X^{(2)} = (X_t^{(2)})_{t\geq 0}$ are given Lévy processes. The research is conducted in Bayesian setting, which means that the prior distribution of the moment of change θ is assumed, i.e. θ is a random variable with a given distribution. The goal is to recognize moment θ upon observations of the process X only. The criterion of optimality is formulated in terms of a probability of false alarm and a mean delay of detection. Then it is reformulated in terms of a posterior probability based on filtration generated by the process X.

The solution of presented problem consists of both optimal stopping time and optimal value function. Methods of solution include connection to a certain free-boundary problem and identification of the infinitesimal generator of the posterior probability process, as well as numerical approach to solve a complex differential equation in one of the models.

The dissertation consists of introduction and three main chapters. Each of them considers different underlying process X. The first one introduces the basic Brownian motion model, which was firstly stated and solved by A.N. Shiryaev. The second main chapter considers more complex jump-diffusion model, in which process X consists of both diffusion and a jump part given by a compound Poisson process. The last chapter considers the most general model. Process X is there a multidimensional jump-diffusion process and the post-change drift rate is a random variable (or vector) from a given distribution.

Besides the theoretical considerations, each of the three main chapters contains the application of the model to the analysis of the force of mortality through the last decades in the Polish population. By the construction of the Generalized Shiryaev-Roberts statistics, the theoretical results are applied to detect the drift change in the mortality data based on the Polish life tables.