

POSITIVE SOLUTIONS TO SOME NONLINEAR ELLIPTIC PROBLEMS

ZEINEB GHARDALLOU

Abstract

The thesis deals with some nonlinear elliptic problems. More precisely, we consider equations

$$Lu \pm \varphi(\cdot, u) = 0, \text{ in the sense of distributions, in } \Omega,$$

where L is a second order elliptic operator with smooth coefficients defined in a Greenian domain $\Omega \subset \mathbb{R}^d$ ($d \geq 3$) such that $L1 \leq 0$ and φ a positive measurable function satisfying some appropriate hypotheses. Using methods of potential theory we describe positive continuous solutions. In particular, we generalize and improve previous results proved in the case of L being the Laplace operator in $\mathbb{R}^d$.

Later on we study entire solutions on the harmonic space NA and on the euclidean space $\mathbb{R}^d$. The corresponding elliptic operator is either the Laplace Beltrami operator in NA or the Laplace operator on $\mathbb{R}^d$. First, we prove sharp estimates of the fundamental solution in NA . Next, we give sufficient and necessary conditions for the existence of bounded or large solutions under the hypothesis that the oscillation $\sup_{|x|=r} \varphi(x, \cdot) - \inf_{|x|=r} \varphi(x, \cdot)$ tends to zero sufficiently fast as r tends to infinity.

Keywords. Nonlinear elliptic problems; Regular domain; Greenian domain; Green function, Thickness and thinness at infinity; Harnack inequality; Harmonic spaces; Damek-Ricci spaces; Heat kernel; Entire solutions; Large solutions.