Asymptotic properties of random walks on the unit grid

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Abstract

We introduce and study a class of random walks defined on the integer lattice $\mathbb{Z}^d$ – a discrete space-and time-counterpart of the symmetric α-stable process in $\mathbb{R}^d$. More precisely we start with the simple random walk (S_n) and build a new random walk $S^\tau_n = S_{\tau_n}$, where (τ_n) is a random walk on the set of natural numbers $\mathbb{N}$ which is independent of S. We call (τ_n) a discrete subordinator. Special attention we pay to subordinators (τ_n) defined by the equation

$$\mathbb{E}\big(e^{-\lambda\tau_n}\big) = \big(1-\psi(1-e^{-\lambda})\big)^n, \quad \lambda \geq 0,$$

where ψ is a Bernstein function. When $\psi(\lambda) = \lambda^{\alpha/2}$, $0 < \alpha < 2$, we call the corresponding random walk S^τ the α-stable random walk and we denote it by S_α. Our choice of the subordinator is strongly motivated by the following functional relation of discrete generators,

$$I - P_\tau = \psi(I-P),$$

where P and P_τ are transition operators of S and S^τ respectively.

We study massive (recurrent) sets with respect to S_α. When $0 < \alpha < 2$ and $d \geq 3$ any coordinate axis in $\mathbb{Z}^d$ is a S_α-non-massive set whereas any cone is S_α-massive. We provide a necessary and sufficient conditions for a thorn to be a massive set. When a thorn is massive we also study the problem of massiveness of its proper subsets. Various examples include subthorns generated by different subsets of the set of primes such as Leitmann and Piatetski-Shapiro primes.

Our results are based on very precise lower bounds for the capacity $\mathrm{Cap}_\alpha(B)$ of certain subsets $B \subset \mathbb{Z}^d$, on the asymptotic formula for the Green function $G_\alpha(x,y)$, and on the Wiener-type criterion of massiveness.