Abstract

In the dissertation we study definable topological dynamics of groups mostly definable in o-minimal expansions of real closed fields. Given an o-minimal expansion of the field of reals $\mathbb{R} = (\mathbb{R}, +, \cdot, <, \ldots)$ and $R \succ \mathbb{R}$ we consider an R-definable group G and the natural action of $G(R)$ on the space $S_{G,ext}(R)$ of external types in G over R.

The first two chapters are devoted to preliminaries.

In the third chapter we prove some results that abstract from the predominant, o-minimal setup. Working over any first-order structure M such that all types over M are definable, we consider an M-definable group G admitting a definable decomposition $G = KH$ in terms of its subgroups K, H with $K \cap H = \{e\}$, such that the group H is definably extremely amenable. We show that certain aspects of the flow $(G(M), S_G(M))$ are explained in terms of the induced flow $(G(M), S_K(M))$, i.e. dynamics of the natural action of G on its subgroup K. Specifically, the results show a homeomorphism of a minimal subflow of $(G(M), S_G(M))$ and a minimal subflow of $(G(M), S_K(M))$, and the existence of an image algebra of strongly generic subsets of $G(M)$ that consists of cylinders over an image algebra of strongly generic subsets of $K(M)$. We also obtain much stronger results assuming that H is a normal factor.

In the fourth chapter we obtain more concrete results for groups definable in an o-minimal setting. We primarily consider $\mathbb{R}$-definable groups that admit a model-theoretic analogue of Iwasawa decomposition: the compact-torsion-free decomposition $G = KH$ with K compact, H torsion-free. For such a group, we provide a partial description of minimal subflows of its universal definable flow, and show that the Ellis group of its universal definable flow is isomorphic as an abstract group to $N_G(H) \cap K(\mathbb{R})$. This negatively answers an early question of Newelski whether (under some nice assumptions) the Ellis group is isomorphic to G/G^{00}. Again, we get stronger results when assuming that H is a normal factor.

We then proceed to consider universal covers of $\mathbb{R}$-definable groups, naturally interpreted in a certain two-sorted structure, and generalize the results on minimal subflows and the Ellis group to this more general context.

Finally, we present certain results on image algebras of strongly generic subsets for R-definable groups, in particular the case of $G = SL(2, -)$.