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ABSTRACT

Dislocations, also called linear defects, are the most important class of imperfections occurring in all crystalline solids like crystals and metals. Their presence has a significant influence on mechanical properties of solids which are highly sensitive to solid perfection. The most relevant ones are diffusion phenomena, plasticity and crystal strength. The reason why dislocations are so essential is that even one defect of a proper type may cause irreversible effects.

The aim of the dissertation is to investigate two models describing evolution of dislocations in crystals. The first model involves a non-linear and non-local equation in one space dimension, where the non-local term is represented by the fractional Laplace operator. For such a problem we study existence of weak solutions and we prove hypercontractivity estimates. Besides, we also study existence and non-existence of self-similar solutions. The second model consists of a system of ordinary differential equations, where each equation describes evolution of exactly one dislocation. Our main goal is to study a long time behaviour of the dynamics of dislocations leading to the creation of so-called walls of dislocations.

In Chapter 1 we give an extended introduction to dislocation theory, where we describe a phenomenon of the motion of dislocations. Moreover, we provide a deeper presentation of the models, which are under the investigation in the dissertation. The main purpose of Chapter 2 is to introduce the fractional Laplace operator occurring in the first model and properties of the fractional derivative. Furthermore, in Chapter 3 we prove existence of weak

solutions, by considering approximating solutions of the regularised problem via the vanishing viscosity method, and study the long time behaviour of the L^p-norms of solutions. In Chapter 4 we study existence of self-similar solutions, and we show the intermediate asymptotics in one particular case. Chapter 5 is devoted to the extinction of solutions of the local counterpart of the first model, where the fractional Laplace operator is replaced by the usual one. Finally, in Chapter 6 we investigate the second model, and we prove the accumulation of dislocations.