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Dear Sir/Madam,

Report on PhD thesis by Jan Dobrowolski 'Groups and rings in some model-theoretic and model-theoretic-motivated contexts

This PhD thesis is in the general area of model theory (a branch of mathematical logic), though aspects of it concern more the general study of topological algebraic structures. It has two separate themes, namely (a) ω-categorical groups and rings (Chapter 1), and (b) profinite and Polish structures (Chapters 2-4), but some of the ideas are common to both (a) and (b). The thesis is an impressive body of work, with many interesting results, often with difficult and original proofs, with techniques from several areas handled with virtuosity. It combines the material in four research papers by the author (three of them already published, two of them joint with Krupiński).

A common theme in (a) and (b) is Shelah's notion of *stable* first order theory. Stability yields a beautiful theory of independence on models of the theory. Recently, the scope (and thereby the applications) of stability theory has been greatly extended, with parts of the theory developed for *simple* theories, *NIP* theories, *rosy* theories, and *NTP2* theories, for example. In a slightly different direction, Hrushovski and Pillay developed in 2011 the notion of a *generically stable type*, and in a similar spirit (but not formally in a model-theoretic context), Krupiński, following earlier work of Newelski, developed stability-like notions of independence in certain topological structures.

(a) **ω-categorical groups and rings.** A countably infinite first order structure is *ω-categorical* if it is determined up to isomorphism by its cardinality and first order theory, or equivalently, if its automorphism group has just finitely many orbits on n-tuples for all n. An extensive body of results on ω-categorical groups and rings emerged in the 1970s and early 1980s. A highlight was the 1979 structure theory for stable ω-categorical groups, due to Baur, Cherlin, and Macintyre, one of the seminal contributions to the rich subject of stable group theory.

In Chapter 1, the two main results are that any ω-categorical generically stable group is nilpotent-by-finite (Theorem 0.2) and any ω-categorical generically stable ring is nilpotent-by-finite (Theorem 0.3). Here, a group is said to be *generically stable* if it has a generically stable global generic type, and has 'finitely satisfiable generics' (fsg). These theorems generalise results of Baur-Cherlin-Macintyre and Baldwin-Rose respectively. The proofs are intricate, with many techniques used and cleverly exploited: the Wilson structure theory for ω-categorical characteristically simple groups; elementary methods from group representation theory; sophisticated recent ideas around generic stability, the fsg condition on a group, connected components and absolute connectedness, and chain conditions on definable subgroups; commutator calculations in nilpotent groups; and various arguments with Jacobson radicals. The work is a powerful indication that much of stable group theory can be extended to the broader class of generically stable groups.

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(b) **Profinite and Polish structures.** Chapter 2, which is mainly algebraic, focusses on the structure of those profinite rings which are *weakly locally finite*, that is, every 1-generated subring is finite. A complete characterisation is given in Theorem 0.14 of semisimple weakly locally finite profinite rings (they are isomorphic as topological rings to a direct product of complete matrix rings over finite fields, just finitely many isomorphism types of matrix rings occurring). This is fairly straightforward, but involves a nice argument, used repeatedly in the thesis, to reduce to the unital case. It is then shown (Theorem 0.15) that in a weakly locally finite profinite ring, the Jacobson radical is nil of finite nilexponent. The proof of the latter employs a beautiful and original argument via Baire category to somehow switch quantifiers.

The remainder of Chapter 2 concerns the notions of *small* profinite ring R (a profinite ring admitting a group of automorphisms which preserves the associated inverse system and has countably many orbits on k-tuples for all k). Smallness is a natural analogue of ω-categoricity, with an appropriate cardinality restriction on the number of orbits on k-tuples (finiteness replaced by countability); thus, much of the material in Chapters 2–4 has a similar flavour to Chapter 1. The conjecture here (Conjecture 0.16) is that every small profinite ring has nilpotent Jacobson radical. This is not proved, but Dobrowolski makes progress in the commutative case (Corollary 2.25) via an intriguing argument combining ring theory with the notion suggested by model theory of 'm-independence'.

Chapter 3 is set in the context of *Polish G-groups*, that is, pairs (H, G) where H is a Polish group and G is a Polish group acting continuously on H; this extends that of a profinite group H equipped with a closed group of automorphisms. Again, (H, G) is *small* if G has countably many orbits on H^k for all $k \in \mathbb{N}$. Dobrowolski gives constructions which answer two published questions of Krupiński. He constructs a small Polish G-group which is not zero-dimensional, making ingenious use of the complete Erdös space. He also constructs a small Polish G-group which has no nm-generic orbit (that is, no orbit which is large in the sense proposed by Krupiński, motivated by model theory but defined topologically). Both constructions are sophisticated and original. He also gives a rather flexible construction of a small Polish group which is 'nm-stable' (in the sense of Krupiński) but is not soluble-by-countable. This shows that without an extra assumption of compactness, obvious analogues of the Baur-Cherlin-Macintyre results do not carry across from ω-categorical groups to small Polish G-groups.

Finally, Chapter 4 tackles the natural-looking question: given a topological group G acting on a set X, where X may be equipped with G-invariant algebraic structure, when is there a (nice) topology on X such that the G-action is continuous, and the topology is compatible with any assumed algebraic structure on X? An elementary answer is given when X has no algebraic structure (Proposition 4.1). When X has a group structure, a finest possible topology on X is described (Theorem 4.3), and likewise, when X has a ring structure, a finest possible topology is described (Theorem 4.10). These results make interesting use of recent work of Bergman. There is further discussion in the context of Polish structures (X, G).

Overall assessment. This body of work constitutes in my opinion a very fine PhD thesis. It uses sophisticated techniques from modern model theory, and serious ideas from group theory, ring theory, elementary representation theory, and general topology – Dobrowolski shows that he can absorb and use a wide range of difficult material. In addition, it shows a great deal of originality and ingenuity in the detailed arguments. Several of the questions answered were posed in significant previous publications, and most of the work will be of interest to contemporary model theorists. Three of the four papers on which the material is based are already published in major international journals, and for the two joint with his supervisor, Krupiński confirms that Dobrowolski made a major contribution. One of these papers builds on a fifth paper, by Dobrowolski and Krupiński, based on work in Dobrowolski's MSc. thesis.

I checked large parts of the work and believe that the mathematics is correct. The writing style is excellent – clear, well-organised, with detail when needed without losing the overview, and with a broader intuition conveyed. The English is near perfect, with very few typographical errors. The thesis indicates that Dobrowolski promises to have a highly successful research career.

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On the basis of this thesis, I very strongly recommend that Dobrowolski be awarded a PhD. Furthermore, I consider that the thesis, in its breadth of technique and subtlety of argument, goes well beyond the expected level of a PhD. I regard it as an outstanding piece of work, with real excellence.

Your sincerely,

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